

MAS125 Calculus II 2006-2007

Problem sheet for exercise class 4

- **Please attend the exercise class to which you have been assigned**
- The instructor will present the starred problems in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online after 5pm Friday.
- The unstarred questions are similar to the ones in this week's coursework.

(*)Problem 1: Draw graphs to show that the n th partial sum of the harmonic series satisfies

$$\ln(n+1) = \int_1^{n+1} \frac{1}{x} dx \leq 1 + \frac{1}{2} + \dots + \frac{1}{n} \leq 1 + \int_1^n \frac{1}{x} dx = 1 + \ln n.$$

Deduce that the harmonic series diverges. Note that there is no empirical evidence for this because the partial sum grows too slowly. If you started with $s_1 = 1$ when the Universe began 13 billion years ago and added a new term every second, how large would the sum be today? Also deduce that the sequence

$$a_n = 1 + \frac{1}{2} + \dots + \frac{1}{n} - \ln n$$

converges. Its limit is called Euler's number and has the value $\gamma = 0.5772\dots$

(*)Problem 2: Use the integral test to show for what values of p the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$$

converges. Show that neither the ratio test nor the root test provides this information.

Problem 3: Use the n th term test or integral test to determine whether the following series converge or diverge:

$$(a) \sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}, \quad (b) \sum_{n=1}^{\infty} n \sin \frac{1}{n}, \quad (c) \sum_{n=1}^{\infty} \frac{8 \tan^{-1} n}{1+n^2}.$$

Problem 4: Use the comparison test and the known behaviour of the p -series to determine if the following series converge or diverge:

$$(a) \sum_{n=3}^{\infty} \frac{1}{\ln(\ln n)}, \quad (b) \sum_{n=1}^{\infty} \frac{10n+1}{n(n+1)(n+2)}, \quad (c) \sum_{n=1}^{\infty} \sin \frac{1}{n}.$$

Problem 5: Use the ratio test or root test to determine whether the following series converge or diverge:

$$(a) \sum_{n=1}^{\infty} \frac{n^{\sqrt{2}}}{2^n}, \quad (b) \sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}, \quad (c) \sum_{n=1}^{\infty} \frac{1.3\dots(2n-1)}{4^n 2^n n!}.$$